WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER TWO 2017
QUESTIONS OF REVIEW 8: Sample Means

Name: ______________________

Wednesday 20th September   Time: 20 minutes   Mark   /20

Calculators assumed.

1. [10 marks – 2, 2, 3 and 3]

Willy Wonka's wonderful chocolate factory produces bars of fudge that have a mean weight of 35.2 grams and a standard deviation of 0.27 grams.

These bars are packed into boxes of 50.

http://3.bp.blogspot.com/-wiligbYaMsk/T_mfjspr3QI/AAAAAAAAAJ0/2jYIrhn3CgI/s1600/Delight+in+Willy+Wonka+Chocolate+Bars.jpg, accessed 14 September 2017

(a) Describe the distribution (type, mean and standard deviation) of the average weight of bars in a box.

Normal (by CLT)

$\bar{x} = 35.2 \quad S_{\bar{x}} = \frac{0.27}{\sqrt{50}} = 0.0382$

(b) What is the probability this average weight is between 35.15 and 35.35 grams?

$P(35.15 \le \bar{X} \le 35.35) = 0.9047$ using Norm CDf

(c) Determine a 95% confidence interval for the total weight of the Wonka fudge bars in a box.

$35.2 \pm \frac{1.96 \times 0.27}{\sqrt{50}}$ is 35.125 to 35.275 g for $\bar{x}$

$\therefore$ box is $\times 50$: 1.756 kg to 1.764 kg

(d) Compare the width of your confidence interval in (c) with the width of a 95% confidence interval for the weight of a display package that contains 20 Wonka fudge bars. Explain.

Width = 8 grams

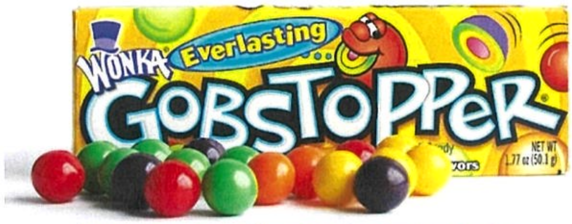
for 20 : $\frac{1.96 \times 0.27}{\sqrt{20}} = 0.118$ $\therefore$ width $= 0.118 \times 2 \times 20 = 4.7$ g

This is narrower because n is less, but off set by a larger s.d.

2. [10 marks – 3, 4 and 3]

10% of the production of Gobstoppers are Orange Outbursts and the Oompa Loompas are putting the Gobstoppers into packs of 30.

They then count 80 packs into each carton for distribution to the wholesalers.



https://www.ruralking.com/media/catalog/product/cache/1/image/9df78eab33525d08d6e5fb8d27136e95/w/o/wonka-s-everlasting-gobstoppers-165-p_ekm_714x300_ekm_.jpg, accessed 14 September 2017

(a) Compare the distributions of the number of Orange Outbursts in a pack with the average number per pack within a carton and highlight all significant differences.

Pack : Number is binomial , $n = 30$ $p = 0.1$

$\mu = 3$ $\sigma = \sqrt{2.7} = 1.6432$

Carton : Ave number is normal $\mu = 3$ but $S_{\bar{x}} = \frac{1.6432}{\sqrt{80}} = 0.1837$

$\therefore$ Different distribution ; much smaller s.d. but same mean

Augustus Gloop is in charge of quality control and prefers large samples to test by eating them all. He finds that, in a carton, the average number of Orange Outbursts per pack is 3.5

(b) Set up a 95% confidence interval for the population mean and decide, with reasons, whether this proportion of Orange Outbursts is consistent with the 10% production figure.

$3.5 \pm \frac{1.96 \times 1.6432}{\sqrt{80}}$ is 3.14 to 3.86

3 is outside this C.I + so it is unlikely to belong ($< 5\%$!)

OR $P(\bar{X} \geq 3.5 \text{ for } \mu = 3) = 3.246 \times 10^{-3}$

Very unlikely !

(c) How many packs should be in a carton so that a 99% confidence interval for the population mean of the average number of Orange Outbursts per pack has width 0.5?

$n = \left(\frac{z \times s}{d}\right)^2 = \left(\frac{2.576 \times 1.6432}{0.25}\right)^2 \simeq 287$